Math 242 Midterm 2

Name: ______________________________

Please circle your section:

Recitation 1	Thurs 12-12:50	TA - Dan Flores
Recitation 2	Thurs 1:30-2:20	TA - Dan Flores
Recitation 3	Tues 9-9:50	TA - Vince Chung
Recitation 4	Tues 12-12:50	TA - Vince Chung
Recitation 5	Wed 9:30-10:20	TA - Lance Ferrer
Recitation 6	Wed 12:30-1:20	TA - Lance Ferrer
Recitation 7	Fri 10:30-11:20	TA - Ikenna Nometa
Recitation 8	Fri 12:30-1:20	TA - Ikenna Nometa
Recitation 9	Fri 9:30-10:20	TA - Dan Flores

Question	Points	Score
1	6	
2	6	
3	10	
4	10	
5	10	
6	6	
7	6	
8	16	
9	30	
Total:	100	

- You may not use notes or calculators on the test.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it).
- The last page is a formula sheet. You are welcome to remove this from the exam.
- Good luck!

1. (6 points) Express the rational function as partial fraction decomposition (to receive full credit, you'll need to put it in the right form and determine the values of the coefficients):

$$\frac{2x^2 + 4x + 1}{x^2 + 2x}$$

2. (6 points) Is the integral $\int_2^{\infty} \frac{1}{x^{3/2}} \mathrm{d}x$ convergent or divergent? Explain your answer (you do not need to evaluate the integral).

3. (10 points) Evaluate the integral $\int \frac{2x+3}{x(x^2+1)} \mathrm{d}x$.

4. (10 points) Evaluate the integral $\int_1^3 \frac{1}{\sqrt{3-x}} \mathrm{d}x$. Be sure to indicate if the integral is improper.

5. (10 points) Suppose you want to approximate the integral $\int_0^3 \sqrt{x+1}\, dx$ with the trapezoid rule so that the error is less than 0.01. How big should you choose n? (See the back page for formulas you may find relevant.)

6. (6 points) Find the limit of the sequence $s_n = \frac{(-1)^n}{2n-1}$

7. (6 points) Explain why the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n+1}}$ diverges.

8. Evaluate the series

(a) (8 points) $\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$

(b) (8 points) $\sum_{n=3}^{\infty} \left(\sin\left(\frac{\pi}{n}\right) - \sin\left(\frac{\pi}{n+1}\right)\right)$.

9. For the following problems, determine if the series is convergent or divergent. Carefully justify each answer.

   (a) (10 points) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

(b) (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n + \sqrt{n}}$

(c) (10 points) $\sum_{n=1}^{\infty} \frac{2 + \cos(n)}{3^n + 1}$

Formula sheet

- Derivatives of inverse trigonometric functions.

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{x\sqrt{x^2-1}}$$

- Trigonometric identities.

$$\sin^2 x + \cos^2 x = 1 \qquad \sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$1 + \tan^2 x = \sec^2 x \qquad \cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$1 + \cot^2 x = \csc^2 x \qquad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$
$$\sin^2 x = \frac{1}{2}(1-\cos(2x))$$
$$\cos^2 x = \frac{1}{2}(1+\cos(2x))$$
$$\sin x\cos x = \frac{1}{2}\sin(2x)$$
$$\sin x\sin y = \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y)$$
$$\cos x\cos y = \frac{1}{2}\cos(x-y) + \frac{1}{2}\cos(x+y)$$
$$\sin x\cos y = \frac{1}{2}\sin(x-y) + \frac{1}{2}\sin(x+y)$$

- Integrals of trigonometric functions.

$$\int \tan x\ dx = \ln|\sec x| + C$$
$$\int \cot x\ dx = \ln|\sin x| + C$$
$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$
$$\int \csc x\ dx = -\ln|\csc x + \cot x| + C$$

- Trapezoidal Rule and Simpson's Rule.

$$T = \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + \ldots + 2y_{n-1} + y_n\right)$$
$$S = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

- Error estimates for Trapezoidal Rule and Simpson's Rule.

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \leq M \text{ for all } x \text{ in } [a,b]$$
$$|E_S| \leq \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \leq M \text{ for all } x \text{ in } [a,b]$$